# Assignment-1 B. Sc. Semester-V (2019-20)

Mathematics Paper- 05(A)

Date of Submission: 13/08/2019 Time: 12:00 to 12:30

Q. 1 Answer the following questions.

1) If (X, d) is a metric space than prove that  $\left(X, \frac{d}{1+d}\right)$  is also a metric space.

2) State and prove Housedroff's principle.

3) Prove that a neighborhood of any point of metric space is an open set in metric space.

4) Is  $\frac{4}{5}$  in cantor set? Justify your answer.

5) Prove that derived set of any any subset of metric space is closed.

Assignment-2 B. Sc. Semester-V (2019-20)

Mathematics Paper- 05(A)

Date of Submission: 27/08/2019 Time: 12:00 to 12:30

**Q. 1** Answer the following questions.

1) Let f be a bounded function defined on [a, b]. P and  $P^*$  are two partition of [a, b] such that  $P^*$  is a refinement partition of P, then  $L(p, f) \le L(P^*, f) \le U(P^*, f) \le U(p, f)$ .

2) State and prove Darboux's theorem.

3) State and prove necessary and sufficient condition for a bounded function f defined on [a, b] to be R-integrable.

4) Prove that bounded monotonic function defined on [a, b] is R-integrable.

5) Find  $\int_0^1 x^2$  using definition of Riemann integrable.

Assignment-3 B. Sc. Semester-V (2019-20)

Mathematics Paper- 05(A)

 Date of Submission: 06/09/2019
 Time: 12:00 to 12:30

**Q. 1** Answer the following questions.

Show that G is a commutative group if (ab)<sup>i</sup> = a<sup>i</sup>b<sup>i</sup>, ∀ a, b ∈ G, for any three conjugative integers.
 Let G be a commutative group. Let a, b ∈ G such that O(a) = m and O(b) = n, then O(ab) = mn if (m, n) = 1.

3) Prove that intersection of two subgroups of a group is again a subgroup.

4) Prove that the set  $A_n$  of all even permutations of  $S_n$   $(n \ge 2)$  is a subgroup of  $S_n$  of order  $\frac{n!}{2}$ .

5) Prove that a normalizer of a group G is a subgroup of G.

Assignment-4 B. Sc. Semester-V (2019-20)

Mathematics Paper- 05(A)

 Date of Submission: 14/09/2019
 Time: 12:00 to 12:30

**Q. 1** Answer the following questions.

1) Prove that a subgroup of index 2 in a group is a normal subgroup.

2) State and prove Cayley's theorem for group.

3) Prove that a subgroup H of a group G is a normal subgroup if and only if

 $(H_a)(H_b) = H(ab), \forall a, b \in G.$ 

4) A subgroup *H* of a group *G* is a normal subgroup if and only if  $aha^{-1} \in H$ ,  $\forall a \in G$ ,  $\forall h \in H$ .

5) Let *H* be a normal subgroup of a group *G*. Then the set  ${}^{G}/_{H}$  of all right cosets of *H* in *G* form a group with respect to the product of cosets of *H* in *G*.